MECE 547 week 11

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Question: For an 6x6 matrix A with a single eigenvalue $\lambda_i = 1$ how many different forms of Jordan forms exists? For each form find characteristic equation, minimal polynomial and ~~null spaces of~~ dimention of null spaces of ~~$(A_i = A - \lambda_i I)$~~ powers of A_i $(A_i = A - \lambda_i I)$. Also show the Jordan form of this matrix $\tilde{A} = P^{-1}AP$ where $\tilde{A}$ is the Jordan form of A. (m(s) → minimal polynomial, d(s) → characteristic equation)

① A single 6x6 Jordan block

$m(s) = (s-1)^6$ → minimal polynomial $\quad d(s) = (s-1)^6$

$\dim(N(A_1^1)) = 1 \xrightarrow{+1} \dim(N(A_1^2)) = 2 \xrightarrow{+1} \dim(N(A_1^3)) = 3$

$\dim(N(A_1^4)) = 4 \xrightarrow{+1} \dim(N(A_1^5)) = 5 \xrightarrow{+1} \dim(N(A_1^6)) = 6$

+1

Always +1 change between null spaces in order so there is only "1" block

$$\tilde{A} = P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}_{6\times 6}$$

→ gives the number of blocks which is "1"

2 – Single 5×5 block
+ Single 1×1 block
total 2 blocks

$m(s) = (s-1)^5$ → biggest block size gives the order of the minimal polynomial

$d(s) = (s-1)^6$

$\dim(N(A_1^1)) = 2$ → there exist 2 blocks

$\dim(N(A_1^2)) = 3$, $\dim(N(A_1^3)) = 4$, $\dim(N(A_1^4)) = 5$

$\dim(N(A_1^5)) = 6$

gives the biggest block size

$$= \left[\begin{array}{ccccc:c} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

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3 – Single 4×4
+ Single 2×2
total 2 blocks

$m(s) = (s-1)^4$

$d(s) = (s-1)^6$

gives the biggest block size

$\dim(N(A_1^1)) = 2$ → there are two blocks

$\dim(N(A_1^2)) = 4$ (Both blocks have size ≥ 2)

$\dim(N(A_1^3)) = 5$ (one block have size ≥ 3)

$\dim(N(A_1^4)) = 6$ (one block have size $= 4$)

$$\tilde{A} = \left[\begin{array}{cccc:cc} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

(4) 1 unit 4x4 block
2 unit 1x1 block
3 blocks in total

$m(s) = (s-1)^4$ maximum block size
$d(s) = (s-1)^6$

$\dim(N(A_1^1)) = 3 \to$ gives total 3 block

+1 change

$\dim(N(A_1^2)) = 4 \to$ ~~gives the number~~ one block have size ≥ 2
due to (+1) change

+1 change

$\dim(N(A_1^3)) = 5 \to$ one block have size ≥ 3
due to (+1) change

+1 change

$\dim(N(A_1^4)) = 6 \to$ one block have size $= 4$
due to (+1) change

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(5) 2 unit 3x3 block
2 blocks in total

$m(s) = (s-1)^3$ maximum block size
$d(s) = (s-1)^6$

+2 change

$\dim(N(A_1)) = 2 \to$ there are two blocks in total
$\dim(N(A_1^2)) = 4 \to$ due to (+2) change both
of these blocks have size ≥ 2

+2 change

$\dim(N(A_1^3)) = 6 \to$ due to (+2) change both
of these blocks have size $= 3$

For 4

$$\tilde{A} = \left[\begin{array}{cccc:cc} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

For 5

$$\tilde{A} = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

(6) 1 → 3x3 block
1 → 2x2 block
1 → 1x1 block
+
3 blocks in total

$m(s) = (s-1)^3$ ← maximum block size is 3

$d(s) = (s-1)^6$

$\dim(N(A_1)) = 3$ → ~~[illegible]~~ number of blocks is 3

2 change

$\dim(N(A_1^2)) = 5$ → number of blocks whose size ≥ 2 is 2 due to (+2) change

+1 change

$\dim(N(A_1^3)) = 6$ → number of blocks whose size $= 3$ is 1 due to (+1) change

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(7) 3 2x2 blocks

total 3 block

$m(s) = (s-1)^2$ max size of the block

$d(s) = (s-1)^6$

+3 change

$\dim(N(A_1')) = 3$ → number of blocks is 3

$\dim(N(A_1^2)) = 6$ → due to (+3) charge the number of blocks having dimention "2" is 3.

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(8) 2 2x2 block

+2 1x1 block

in total 4 blocks

$m(s) = (s-1)^2$ max block size

$d(s) = (s-1)^6$

+2 change

$\dim(N(A_1')) = 4$ → there are 4 blocks

$\dim(N(A_1^2)) = 6$ → due to (+2) chang then number of blocks having dimention "2" is 2.

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(g)

$$\begin{array}{lll} 1 & 2\times 2 & \text{block} \\ + \; 4 & 1\times 1 & \text{block} \\ \hline 5 & \text{blocks} & \end{array}$$

$m(s) = (s-1)^2$ ← max block size

$d(s) = (s-1)^6$

$\dim(N(A_1^1)) = 5 \to$ there are 5 blocks

+1 change

$\dim(N(A_1^2)) = 6 \to$ due to $(+1)$ change the number of blocks whose size is 2 is 1.

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(h)

$$\begin{array}{lll} + \; 6 & 1\times 1 & \text{block} \\ \hline 6 & \text{blocks in total} & \end{array}$$

$m(s) = (s-1)^1$ ← maximum block size is "1"

$d(s) = (s-1)^6$

$\dim(N(A_1^1)) = 6 \to$ there are 6 blocks

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \to \text{totally diagonal matrix}$$